Spatial coherence

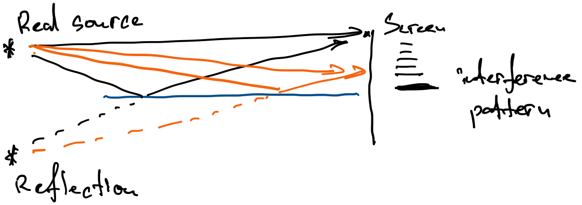
Here we will try to answer how the e/m radiation source dimensions influence visibility V of interference pattern. This is directly connected to such property as spatial coherence.

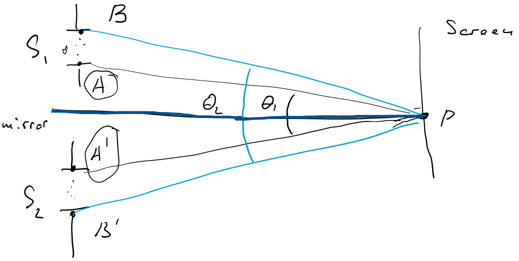
In this chapter, we will consider the light being monochromatic.

How dimensions of the light source can influence interference pattern?

① Qualitative approach

Lloyd's mirror



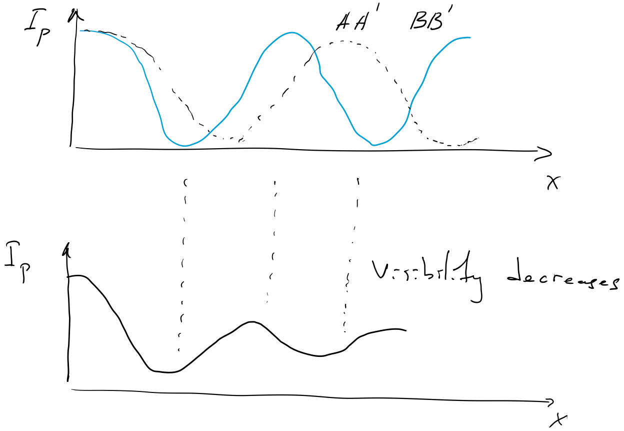


In experiment with Lloyd's mirror if we have source A, A' will be coherent to this source.

$\Delta x = \frac{\lambda}{\theta}$ for A and A' the angle is θ_1

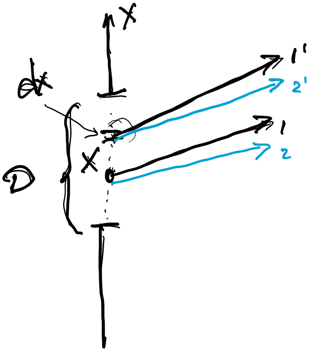
for B and B' the angle is θ_2

The resultant interference pattern:



Our next goal is to quantitatively connect source dimensions with visibility V.

② Quantitative approach



We will take a source as slit D. We can imagine it consists of many small sources dx. This source is at x from slit center.

This source dx emits rays 1' and 2' that are in infinity overlap at point P on the screen. Let's **calculate** such interference pattern.

We will assume dI is an intensity of one beam from source dx.

$$dI = \frac{I_0}{D} dx$$

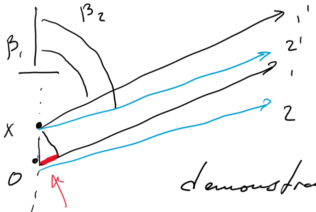
I_0 - intensity of the whole source.

$$dI_P = 2\frac{I_0 \, dx}{D}\left(1+\cos k\Delta(x)\right)$$

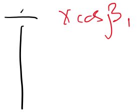
$$\Delta(x) = \Delta_0 + x(\cos\beta_1 - \cos\beta_2)$$

1) Δ_0 is optical path difference between 1 and 2

2) $x(\cos\beta_1 - \cos\beta_2)$ will come from optical path difference as dx is at distance x from center, i.e. path difference between 1 and 1' or 2 and 2'.



One term Δ_0 will be fixed, but second term will directly demonstrate the influence of



source dimensions.

Hence,

$$dI_p = 2\,\frac{I_0\,dx}{D}\left(1+\cos k\Delta(x)\right)$$

$$I_p = \frac{2I_0}{D}\int_{-\frac{D}{2}}^{\frac{D}{2}}\left[1+\cos k\Delta(x)\right]dx =$$

$$= 2I_0\left\{1+\frac{1}{D}\int_{-\frac{D}{2}}^{\frac{D}{2}}\cos\left(k\Delta_0+\gamma x\right)dx\right\}$$

$$\gamma = k\left(\cos\beta_1-\cos\beta_2\right)$$

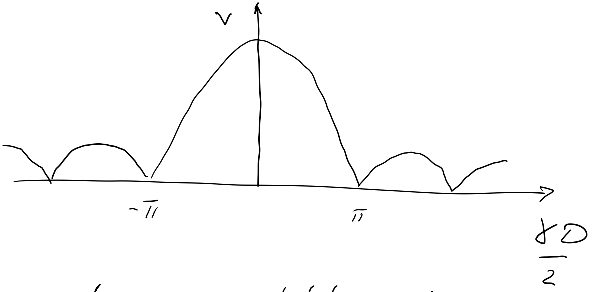
$$I_p = 2I_0\left[1+\frac{\sin\frac{\gamma D}{2}}{\frac{\gamma D}{2}}\cos k\Delta_0\right]$$

$$I_p = 2I_0\left[1+C(D)\cos k\Delta_0\right]$$

$$C(D) = \frac{\sin\frac{\gamma D}{2}}{\frac{\gamma D}{2}}$$

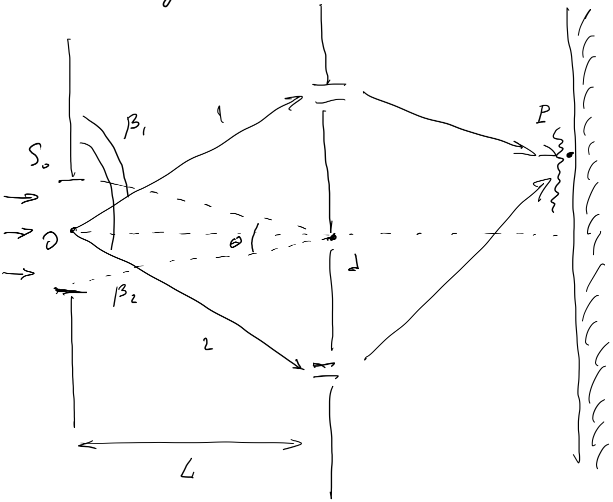
Similar to time coherence:

$$V = |C(D)|$$



That means visibility V decreases with increase of light source dimension.

Let's see how it can be nicely shown in Young's experiments.



Young has shown that interference pattern will be at P depending on the dimension D.

$$\cos\beta_1 = \frac{d}{2L} \qquad \cos\beta_2 = -\frac{d}{2L}$$

$$\gamma = k(\cos\beta_1 - \cos\beta_2) = \frac{kd}{L}$$

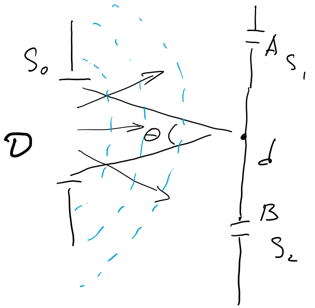
$$V = \left| \frac{\sin \frac{k \cdot d \cdot D}{2L}}{\frac{kdD}{2L}} \right| = \left| \frac{\sin \frac{\pi d D}{\lambda L}}{\frac{\pi d D}{\lambda L}} \right| =$$

$$= \left| \frac{\sin \frac{\pi d \theta}{\lambda}}{\frac{\pi d \theta}{\lambda}} \right| \qquad \Theta \approx \frac{D}{L}$$

$V = 0$, when $\frac{\pi d \theta}{\lambda} = \pi$

$d_c = \frac{\lambda}{\theta}$ → coherence radius

What does it mean?



Oscillations at points A and B will be coherent if $d < d_c$. Slits A and B in fact probe coherence of the source at certain distance or angle of observation.

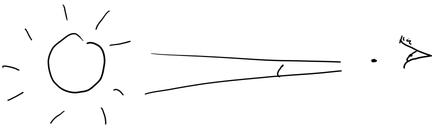
For example, if we have point source, then D is very small and d_c is infinitly large. That means we can take any two slits and will observe interference on the screen.

Let's see Young's genius.

He used Sun light.

$$\Theta_\odot = 30' \approx 0.01 \text{ rad}$$

$$d_c = \frac{\lambda}{\Theta} \approx 50 \mu m$$

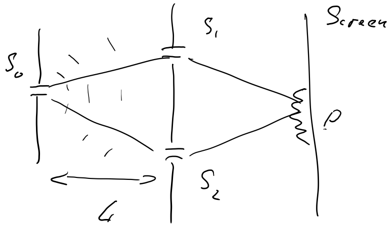


Young realized to use slit S_0

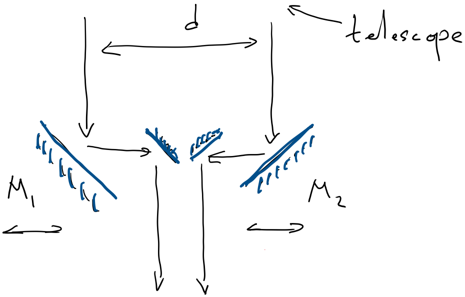
$D = 1 mm$

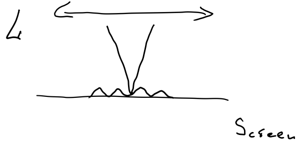
$L = 1 m$

Calculate



Stellar interferometer by Michelson





Imagine, we have interstellar object.

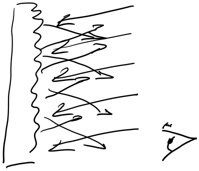
Our goal is to see how coherent it is. It will allow us to measure angular dimension of this star.

$$\left.\begin{array}{l} d \text{ is changing} \\ d_c = \dfrac{\lambda}{\Theta} \end{array}\right\} \Rightarrow \begin{array}{l} \text{Once interference picture disappears} \\ d = d_c \Rightarrow \Theta \end{array}$$

He took α-Orion - Betelgeuse

$\Theta_{Betelgeuse} = 0.047'' \longrightarrow$ 300 Suns.

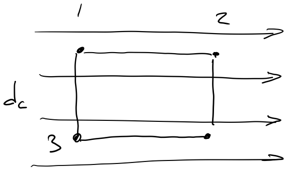
Laser speckles



Laser light has large time and spatial coherence.

For cw laser

$$\ell_c = \frac{\lambda^2}{\Delta\lambda} \longrightarrow \text{meters}$$



Laser beam is almost perfectly collinear beam that is radiated by infinitly far away point source.

d